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Abstracts

Hansjoerg Albrecher (University of Lausanne)

Level crossing identities under discrete observation with applications in insurance

This talk deals with the effects of randomizing observation periods on exit probabilities and related quantities of stochastic processes. It turns out that observation of the process at epochs of an independent Poisson process leads to a number of strikingly simple analogues of classical fluctuation identities. Applications in the context of collective insurance risk models are discussed.

Philippe Berthet (Toulouse Mathematics Institute)

Weak and strong uniform convergence of some random surfaces

We introduce several kinds of surfaces built from an i.i.d. sample in $\mathbb{R}^d$ with law P. Each surface is indexed by the unit sphere and depends on a directional projection rule, a mass index α and an arbitrary point O. No density is required for P thus allowing sparse or low dimensional supports. Moving the observer O give clues about the mass localization of P. These subjective surfaces have many applications in spatial statistics and induce new depth fields defined as random vector fields, allow goodness of fit tests and mass concentration type data analysis procedures. Letting α increase to 1 leads to notions of multivariate extremes and trimming. When the dimension d is increasing with n, they can be used to test or compare samples of processes.

The first kind of surfaces is quantile surfaces [1] and their generalizations. Surface points correspond to the quantile of order α of the projection of the sample on a line containing O. The nature of the projection leads to several types of closed surfaces all satisfying uniform laws of the iterated logarithm, uniform central limit theorems with rate $n^{-1/2}$ and explicit limiting covariance, then a Brownian strong approximations driven from [4]. Our main result is a general Bahadur-Kiefer strong approximation with a dimension free rate $n^{-1/4}$. Since for orthogonal projections O plays no role we can derive the weak and strong limiting behaviour of the so-called Tuckey contours. The second type of surfaces are built from directional empirical argmins such as the shortest bandwidth with mass α [2] or minimum volume generalizations. The convergence of these surfaces is again uniform in direction parameters and α but the dimension free first order rates are $n^{-1/3}$ with a non Gaussian limiting process as for the univariate version [3].

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Umut Can (University of Amsterdam)

Asymptotically distribution-free goodness-of-fit testing for tail copulas

Let $(X_1, Y_1), \ldots, (X_n, Y_n)$ be an i.i.d. sample from a bivariate distribution function that lies in the maximum domain of attraction of an extreme value distribution. The asymptotic joint distribution of the standardized component-wise maxima $\max(X_i)$ and $\max(Y_i)$ is then characterized by the marginal extreme value indices and the tail copula R. The extreme value indices specify the asymptotic marginal distributions of the standardized maxima, and the tail copula specifies the dependence structure. We propose a procedure for constructing asymptotically distribution-free goodness-of-fit tests for the tail copula R. The procedure is based on a transformation of a suitable empirical process derived from a semi-parametric estimator of R. The transformed empirical process converges weakly to a standard Wiener process, paving the way for a multitude of asymptotically distribution-free goodness-of-fit tests. We also extend our results to the m-variate ($m > 2$) case. In a simulation study we show that the limit theorems provide good approximations for finite samples and that tests based on the transformed empirical process have high power.

John Einmahl (Tilburg University)

Statistics of heteroscedastic extremes

We extend classical extreme value theory to non-identically distributed observations. When the tails of the distribution are proportional much of extreme value statistics remains valid. The proportionality function for the tails can be estimated nonparametrically along with the (common) extreme value index. For a positive extreme value index, joint asymptotic normality of both estimators is shown; they are asymptotically independent. We also establish asymptotic normality of a forecasted high quantile and develop tests for the proportionality function and for the validity of the model. A main tool is the weak convergence of a weighted sequential tail empirical process.

Joint work with Laurens de Haan and Chen Zhou.

Hans-Jürgen Engelbert (Friedrich Schiller University of Jena)

On the chaotic representation property of certain families of martingales

Let $T > 0$ be a finite time horizon, $(\Omega, \mathscr{F}, \mathbf{P})$ a complete probability space and $\mathbb{F} = (\mathscr{F}_t)_{t\in[0,T]}$ a filtration satisfying the usual conditions. By $\mathscr{H}^2 = \mathscr{H}^2(\mathbb{F})$ we denote the Hilbert space of square integrable martingales X on $[0, T]$ with respect to $\mathbb{F}$. In the present talk we consider families $\mathscr{X}$ of martingales contained in $\mathscr{H}^2$ starting from zero. We assume that for every $X, Y \in \mathscr{X}$ the predictable covariation $\langle X, Y\rangle$ is deterministic. We can therefore introduce, for every $n \in \mathbb{N}$, the space $\mathscr{J}_n(T)$ of terminal values of n-fold iterated stochastic integrals generated by the family $\mathscr{X}$. The spaces $\mathscr{J}_n(T)$ and $\mathscr{J}_m(T)$ are orthogonal in $L^2(\Omega, \mathscr{F}, \mathbb{P})$ for every $n \neq m$. We denote by $\mathscr{J}(T)$ the closure in $L^2(\Omega, \mathscr{F}^{\mathscr{X}}, \mathbb{P})$ of the l! inear hul l of $\mathbb{R} \cup \bigcup_{n=1}^{\infty} \mathscr{J}_n(T)$. We say that $\mathscr{X}$ possesses the *chaotic representation property* (CRP) if $\mathscr{J}(T) = L^2(\Omega, \mathscr{F}^{\mathscr{X}}, \mathbb{P})$. If the family $\mathscr{X}$ is orthogonal then the CRP amounts to saying that we have the orthogonal decomposition

$$L^2(\Omega, \mathscr{F}^{\mathscr{X}}, \mathbb{P}) = \mathbb{R} \oplus \bigoplus_{n=1}^{\infty} \mathscr{J}_n(T)\,. \tag{1}$$

If $\mathscr{X} = \{W\}$, W Brownian motion, this is just the chaos expansion of Itô (1951). We call $\mathscr{X}$ compensated-covariation stable if for all martingales $X, Y \in \mathscr{X}$ the martingale $([X,Y] - \langle X,Y \rangle, \mathbb{F})$ again belongs to $\mathscr{X}$ where $[X,Y]$ denotes the covariation of X and Y. The main result is the following

Theorem 1. *Suppose that the following conditions are satisfied:*
1) *The polynomials generated by the family* $\{X_t : \; t \geq 0, \; X \in \mathscr{X}\}$ *belong to* $L^2(\Omega, \mathscr{F}^{\mathscr{X}}, \mathbb{P})$ *and are dense;* **2)** $\mathscr{X}$ *is compensated-covariation stable; and* **3)** $\langle X, Y \rangle$ *is deterministic for all* $X, Y \in \mathscr{X}$. *Then* $\mathscr{X}$ *possesses the CRP.*

Under the conditions of Theorem 1, as a consequence of the CRP, the predictable representation property (PRP) of $\mathscr{X}$ with respect to the filtration $\mathbb{F}^{\mathscr{X}}$ is satisfied, too. As an important application, we shall construct families $\mathscr{X}$ of martingales related with Lévy processes L satisfying the CRP with respect to $\mathbb{F}^L$. The chaos decomposition (1) gives another representation of Itô's chaos expansion in terms of *multiple* Itô integrals introduced by Itô (1956). These results also include the CRP for the orthogonalized Teugels martingales which was studied by Nualart and Schoutens (2000).

Joint work with Paolo Di Tella (TU Dresden, Germany).

Dietmar Ferger (Technische Universität Dresden)

On the convergence of arginf-sets and infimizing points of multivariate cadlag stochastic processes

Let $X_n = \{X_n(t) : t \in \mathbb{R}^d\}, n \geq 1$, be a sequence of stochastic processes defined on some complete probability space $(\Omega, \mathcal{A}, \mathbb{P})$ with trajectories in the space $D = D(\mathbb{R}^d)$ consisting of all functions $f : \mathbb{R}^d \to \mathbb{R}$ which are *continuous from above* with *limits from below* as specified by Lagodowski and Rychlik (1986). They show that D endowed with the *Skorokhod-metric* s is a complete separable metric space and that the pertaining Borel-σ algebra $\mathcal{D}$ is generated by the the projections. Thus each X_n can be considered as a random variable $X_n : (\Omega, \mathcal{A}) \to (D, \mathcal{D})$. In particular, if $A(X_n)$ denotes the set of all infimizing points of X_n, then $A(X_n)$ is a *random closed set* in $\mathbb{R}^d$. More precisely, let $\mathcal{F}$ be the collection of all closed subsets in $\mathbb{R}^d$ equipped with the *Fell-topology* τ_{Fell} and the corresponding Borel-σ algebra $\mathcal{B}_{Fell}$. Then $A(X_n) : (\Omega, \mathcal{A}) \to (\mathcal{F}, \mathcal{B}_{Fell})$ is measurable. A coarser topology on $\mathcal{F}$ is given by the *miss-topology* τ_{miss}, which is also known as *upper Fell-topology*.

We prove that if $X_n \xrightarrow{\mathcal{L}} X$ in (D, s) then $A(X_n) \xrightarrow{\mathcal{L}} A(X)$ in $(\mathcal{F}, \tau_{miss})$. If in addition $(A(X_n))$ is *stochastically bounded* then we obtain *quasi-convergence in distribution* and give a precise characterization. Finally, if furthermore $A(X)$ is a singleton a.s.

then $A(X_n) \xrightarrow{\mathcal{L}} A(X)$ in $(\mathcal{F}, \tau_{Fell})$. In statistics one is interested in *measurable selections* ξ_n. These are random variables with $\xi_n \in A(X_n)$ a.s. Here, $X_n \xrightarrow{\mathcal{L}} X$ in (D, s) entails

$$\limsup_{n\to\infty} \mathbb{P}(\xi_n \in K) \leq T(K) := \mathbb{P}(A(X) \cap K \neq \emptyset) \quad \text{for all compact } K \subseteq \mathbb{R}^d.$$

Under the additional requirement that (ξ_n) is *stochastically bounded* this can be sharpened to hold even for all **closed** subsets K.

The set-function T (extended on the Borel-sets $\mathcal{B}$ of $\mathbb{R}^d$) is called *Choquet-capacity* and in general T needs not to be a probability measure on $\mathcal{B}$. However, we show that T in fact is a probability measure if and only if $A(X) = \{\xi\}$ is a singleton a.s. Thus in this case the Potmanteau-Theorem guarantees that $\xi_n \xrightarrow{\mathcal{L}} \xi$ in $\mathbb{R}^d$.

Eva Ferreira (University of the Basque Country)

The influence of ability distribution on glass ceiling effects

We consider selection processes that choose the best candidates in a hierarchical organization and explore glass ceiling effects. The context is different from [1] and [2], where the abilities are unobservable. Provided there is a constant gender bias equal for different levels of a hierarchy, we show that the explanation for the glass ceiling effect does not need to rely on an exogenous larger discrimination at the top levels, but on the characteristics of the selection process and the population. In fact, the shape of the ability distribution tail and the hierarchy structure are crucial for this effect to appear. To illustrate the influence of both characteristics, we study the performance of different types of glass ceilings under Pareto and Weibull families, considering different classes of hierarchies. These results are consistent with the mixed empirical evidence that has found glass ceilings only in some contexts.

Joint work with Maria Paz Espinosa.

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Irène Gijbels (KU Leuven)

Estimation of conditional, partial and average copulas and association measures

Classical examples of association measures between two random variables include Pearson's correlation coefficient, Kendall's tau and Spearman's rho. For the situation where another variable(s) influences the dependence between the pair, so-called partial association measures, such as a partial Pearson's correlation coefficient and a partial Kendall's tau, have been proposed in the 1940's. In recent years conditional association measures have been studied, such as a conditional Kendall's tau. Such an association measure can be expressed in terms of a conditional copula. See for example [2] and [3].

Even in case the dependence structure between two variables is influenced by a third variable, we still want to be able to summarize the dependence structure of the pair by one single number, taking into account the additional influence. In this paper we discuss two different ways to do so, leading to the study of partial and average copulas (see for example [1]) and corresponding association measures. We study conditional, partial and average copulas and association measures, discuss non- and semiparametric estimation of these, and investigate their asymptotic behaviour. Examples are given to illustrate the use of the concepts and methods.

Joint work with Marek Omelka and Noël Veraverbeke.

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Shota Gugushvili (Leiden University)

A non-parametric Bayesian approach to decompounding from high frequency data

Given a sample from a discretely observed compound Poisson process, we consider non-parametric estimation of the density f_0 of its jump sizes, as well as of its intensity λ_0. We take a Bayesian approach to the problem and specify the prior on f_0 as the Dirichlet location mixture of normal densities. An independent prior for λ_0 is assumed to be compactly supported and possess a positive density with respect to the Lebesgue measure. We show that under suitable assumptions the posterior contracts around the pair (λ_0, f_0) at essentially (up to a logarithmic factor) the $\sqrt{n\Delta}$-rate, where n is the number of observations and Δ is the mesh size at which the process is sampled. The emphasis is on high frequency data, $\Delta \to 0$, but the obtained results are also valid for fixed Δ. In either case we assume that $n\Delta \to \infty$. Our main result implies existence of Bayesian point estimates converging (in the frequentist sense, in probability) to (λ_0, f_0) at the same rate. Simulations complement the theory.

Joint work with Frank van der Meulen and Peter Spreij.

Estate Khmaladze (Victoria University of Wellington)

On Brownian motions, Brownian bridges and unitary operators

We will argue that there are surprisingly many different Brownian bridges, some of them – familiar, some of them – less familiar. Some of these Brownian bridges are very close to Brownian motion. Somewhat loosely speaking, we show that all the bridges can be conveniently mapped onto each other, and, hence, to one "standard" bridge. As a consequence of this, we obtain a unified theory of distribution-free testing in $\mathbb{R}^d$, both for discrete and continuous cases, and for simple and parametric hypotheses.

Consider $w_F(\phi), \phi \in L_2(F)$, a function-parametric F-Brownian motion,

$$w_F(\phi) = \int_{y\in\mathbb{R}^d} \phi(y) dw_F(y),$$

which is a linear functional in ϕ and for each ϕ is a Gaussian random variable with mean 0 and variance $\|\phi\|_F^2$. Denote $v_{F,q}(\phi)$ the function-parametric F-Brownian bridge, defined as a linear transformation of w_F:

$$v_{F,q}(\phi) = w_F(\phi) - \langle \phi, q\rangle_F w_F(q), \quad \|q\|_F = 1, \tag{2}$$

where the case when q is the function identically equal to 1 is very dominant – for many decades we spoke of no other bridges. These bridges appear as limiting objects for empirical processes, associated with samples of n i.i.d.(F) random variables. However, different choices of q emerged, without much recognition, from various testing problems and led to very interesting "bridges". Also the situations with vector-function q are very common. The *q-projected F-Brownian motion* could be a possible alternative term for $v_{F,q}$.

For different distributions, F and G, $v_{F,q}$ and $v_{G,q}$ are associated with different testing problems. We rarely, if ever, discussed the connections between the two. However, the following proposition shows what is possible:

Theorem. *If distributions F and G are equivalent, i.e. if $l^2(x) = dG(x)/dF(x))$ is positive F-a.e., then the process with differential*

$$\begin{aligned} v_{G,1}(dx) =& l(x)\, v_{F,1}(dx) - \int_{y\in\mathbb{R}^d} l(y)\, v_{F,1}(dy) \times \\ & \frac{1}{1 - \int_{y\in\mathbb{R}^d} l(y) dF(y)} [l^2(x) - l(x)] f(x)\, dx \end{aligned} \tag{3}$$

is G-Brownian bridge and the transformation is one-to-one.

Therefore, the problem of testing for F and testing for G can be mapped into each other. In particular, we have the following corollary:

Corollary. *Suppose F is an absolutely continuous distribution on $[0,1]^d$, which has a.e. positive density f. Then the process $u = \{u(x), x \in [0,1]^d\}$ with the differential*

$$u(dx) = \frac{1}{\sqrt{f(x)}} v_{F,1}(dx) - \frac{1 - \sqrt{f(x)}}{1 - \int_{[0,1]^d} \sqrt{f(y)} dy} \int_{[0,1]^d} \frac{1}{\sqrt{f(y)}} v_{F,1}(dy)\, dx \tag{4}$$

is the standard Brownian bridge.

The following construction is behind these and other statements of [1]:

$$v_{G,r}(\psi) = v_{F,q}(K\psi), \quad \psi \in L_2(G),$$

where K is the unitary operator from $L_2(G)$ to $L_2(F)$, which is remarkably simple and will be described in the talk. Relatively long row of other statements follow from this construction.

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Roger Koenker (University of Illinois at Urbana-Champaign)

Frailty, profile likelihood and mortality

Unobserved heterogeneity is an increasingly common feature of statistical survival analysis where it is often referred to as frailty. Parametric mixture models are frequently used to capture these effects, but it is sometimes desirable to consider nonparametric mixture models as well. We illustrate the latter approach with a re-analysis of a well-known large scale medfly mortality study. Recent developments in convex optimization are exploited to expand the applicability of the Kiefer-Wolfowitz nonparametric maximum likelihood estimator for mixture models. Some resulting problems of profile likelihood will also be addressed.

Joint work with Jiaying Gu (University of Toronto).

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Hira L. Koul (Michigan State University)

Residual empirical processes

This talk will discuss current state of residual empirical processes in regression and autoregressive conditionally heteroscedastic (ARCH) models, and their usefulness in advancing statistical inference in these models. In particular an asymptotically distribution free test based on Khmaladze martingale transformation of a certain weighted residual empirical process for fitting an error distribution in ARCH models will be discussed. A part of this talk is based on the paper by Koul and Zhu on 'Goodness-of-fit testing of error distribution in nonparametric ARCH(1) models', available online at the address http://dx.doi.org/10.1016/j.jmva.2015.02.009.

Roger J.A. Laeven (University of Amsterdam & Eurandom)

Return risk measurement: Orlicz-type measures of risk

We provide an axiomatic foundation of Orlicz measures of risk in terms of properties of their acceptance sets, by exploiting their natural correspondence with shortfall risk (Föllmer and Schied, 2004). We explicate that, contrary to common use of *monetary risk measures*, which measures the risk of a financial position by assessing the stochastic nature of its *monetary* value, Orlicz measures of risk assess the stochastic nature of *returns*: they are *return risk measures*.

This axiomatic foundation of Orlicz measures of risk naturally leads to several *robust* generalizations, obtained by generalizing expected utility to ambiguity averse preferences such as variational (Maccheroni et al., 2006) and homothetic preferences (Cerreia-Vioglio et al., 2011, Chateauneuf and Faro, 2010, Laeven and Stadje, 2013). We also consider the case of ambiguity (multiplicity) over the Young function Φ in the definition of the Orlicz measure of risk and the case of a state-dependent Φ leading to *Musielak-Orlicz measures of risk*. From a purely mathematical point of view, the resulting functionals can in a unified way be seen as suprema of Orlicz norms on a suitable rearrangement-invariant Banach space.

We study the properties of these *robust Orlicz measures of risk* and analyze and provide dual representations of their optimized translation invariant extensions (Rockafellar and Uryasev, 2000, Rockafellar, Uryasev and Zabarankin, 2008).

Joint work with Fabio Bellini and Emanuela Rosazza Gianin.

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Tapabrata (Taps) Maiti (Michigan State University)

Functional mixed models for small area estimation

Functional data analysis has become an important area of research due to its ability of handling high dimensional and complex data structure. However, the development is limited in the context of linear mixed effect models and in particular for small area estimation. The linear mixed effect models are the backbone of small area estimation. In this article, we consider area level data, and fit a varying coefficient linear mixed effect model where the varying coefficients are semi-parametrically modeled via B-splines. We propose a method of estimating the fixed effect parameters and consider prediction of random effects that can be implemented using standard softwares. For measuring prediction uncertainties, we derive an analytical expression for the mean squared errors, and propose a method of estimating the mean squared errors. The procedure is illustrated via a real data example, and operating characteristics of the method are judged using finite sample simulation studies.

Joint work with Samiran Sinha and Ping-Shou Zhong.

Badri Mamporia (Georgian Technical University)

The problem of decomposability in development of the stochastic differential equations in a Banach space

First results on infinite dimensional stochastic differential equations started to appear in the mid 1960s. Special interest to this direction is observed after the eighties of the last century. The monograph of K. Ito [1] is devoted to the theory. However, after, the activity was relatively decreased by the reason that the traditional methods of developing the theory do not work in arbitrary Banach spaces. Some results were obtained in UMD Banach spaces. It should be noted that this class of Banach spaces is very narrow – it is a subclass of reflexive Banach spaces. The main problem to study the stochastic differential equations in infinite dimensional spaces is the definition of the Ito stochastic integral. We define the generalized stochastic integral for a wide class of non-anticipating random processes which is a generalized random element (GRE), and if there exists the corresponding random element, that is, if this GRE is decomposable by the random element, then we say that this random element is the stochastic integral. Thus, the problem of existence of the stochastic integral is reduced to the well known problem of decomposability of the GRE. Another problem to develop the stochastic differential equation in a Banach space is to estimate the stochastic integral, which is impossible by the traditional methods. We introduce the space of GREs. It is possible to use the traditional methods in this space. Afterwards, from the main stochastic differential equation in a Banach space we obtain the equation for a generalized random process and get the solution as a generalized random process; if this generalized random process is decomposable, then the obtained Banach space valued random process will be the solution of the main equation. Thus, we reduced, as well, the problem of existence of the solution to the problem of decomposability of the generalized random process.

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Michael Mania (Razmadze Matematical Institute)

On regularity of dynamic value function related to utility maximization problem

We study regularity properties of the dynamic value function and of the optimal solution to the utility maximization problem for utility functions defined on the whole real line. These properties we need to show that the value function satisfies a corresponding backward stochastic partial differential equation. In particular, we give conditions on the utility function and on the market model when this equation admits a solution.

Joint work with Revaz Tevzadze.

Gennadi Martynov (IITP RAS & HSE, Moscow, Russia)

Cramér-von Mises test for Gaussian measure in Hilbert Space

In statistical applications it is often assumed that an observed random process is Gaussian. Often, this assumption is accepted in practice without an adequate analysis. We propose here a generalization of the Cramér-von Mises test for testing the null hypothesis that an observed random process on the interval $[0, 1]$ is a mean zero Gaussian process with specified covariance function. We assume the alternative random processes are all the other Gaussian processes, together with all non-Gaussian processes. The test statistic is based on the finite number of the process observations. In fact, we consider the more general problem of testing the hypothesis that the distribution in a separable Hilbert space is Gaussian. To test this simple hypothesis we propose a Cramér-von Mises test based on an infinite-dimensional analogue of the empirical process. The proposed test is asymptotically consistent against all alternatives. We also provide a method for computing the critical values of our test statistic. It was calculated the exact critical values of the test. The same theory also applies to the problem of testing multivariate uniformity over a high-dimensional hypercube. This investigation is based upon previous joint work by Paul Deheuvels and the author [2]. The first idea of this method have been proposed by the author [3], [4]. The main results are presented in the paper [1].

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Ian McKeague (Columbia University)

Stein's method and the many-worlds interpretation of quantum mechanics

It has been conjectured that quantum effects arise from the interaction of finitely many classical "worlds". The wave function is then thought to be recoverable from observations of particles in these worlds, without knowing the world from which any particular observation originates. In this talk we discuss how Stein's method (typically used to obtain rates of convergence in central limit theorems) can be used to obtain such a result as the number of worlds goes to infinity. We examine the ground-state configuration of a parabolic potential well, and show that the particle configuration is asymptotically Gaussian, thus matching the stationary ground-state solution of Schroedinger's equation. Further, we construct a sequential bootstrap of the mean particle location and show that it converges to an Ornstein-Uhlenbeck process, the time-dependent ground-state solution of Schroedinger's equation in this setting.

Robert Mnatsakanov (West Virginia University)

Approximation of ruin probability and aggregate claim size distribution in the classical risk model

We consider the problem of recovering the ultimate ruin probability and compound distribution in the classical Poisson risk model ([1]-[3]). The uniform convergence of corresponding approximations based on the finite number of values of the Laplace transform of claim size distribution are derived.

The asymptotic behavior of proposed approximations are illustrated by means of a simulation study. The empirical counterparts of approximations demonstrate sufficiently accurate estimation of underlying functions. The comparisons with other approximations based on the maximum entropy method and on a fixed Talbot algorithm are carried out via tables and plots in several typical examples.

Joint work with Artak Hakobyan and Khachatur Sarkisian.

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Goran Peskir (The University of Manchester)

Quickest detection problems for Bessel processes

Imagine the motion of a Brownian particle that initially takes place in a two-dimensional plane and then after some random/unobservable time continues in the three-dimensional space. Assuming that only the distance of the particle to the origin is being observed, the problem is to detect the time at which the particle departs from the plane as accurately as possible. I will present some recent results on this problem and discuss its solution.

Joint work with Peter Johnson (The University of Manchester).

Holger Rootzén (Chalmers University)

Multivariate Peaks over Thresholds modelling

Extreme episodes which develop in time and/or space are crucial for risk handling in climate science, environmental science, civil engineering, finance.... This talk is an overview over ongoing work on multivariate Peaks over Thresholds modeling of extreme episodes. Topics include new parametric models; relations between multivariate generalized Pareto distribution and point process limits of extreme values; connection between statistical methods based on multivariate point processes models and methods based on multivariate generalized Pareto models; spatial extreme value models with nuggets; simulation of multivariate generalized Pareto distributions; and likelihood inference.

Joint work with Anna Kiriliouk, Johan Segers and Jennifer Wadsworth.

Johannes Schmidt-Hieber (Leiden University)

Non-parametric Le Cam theory: overview and recent results

Suppose we are interested in estimation of a parameter in a given statistical model. Under various circumstances there exists a simpler model that contains the same information about the unknown parameter. This concept of sufficiency is useful for the theoretical understanding of the estimation problem and leads to a more unified view on statistics. For non-parametric problems, that is, if we consider estimation of complex objects such as functions, the notion of sufficiency becomes too strong. In the late '90s, it was shown for two non-parametric models, that sufficiency holds in an asymptotic sense as introduced earlier by Le Cam. Since then, few other results of this type could be established. In the first part of the talk we review the development of this field. In the second part, we explain a recent result on asymptotic equivalence for regression under dependent noise.

Martin Schweizer (ETH Zürich)

A new stochastic Fubini theorem

The classic stochastic Fubini theorem says that if one stochastically integrates with respect to a semimartingale S an $\eta(dz)$-mixture of z-parametrised integrands ψ^z, the result is just the $\eta(dz)$-mixture of the individual z-parametrised stochastic integrals $\int \psi^z \, dS$. But if one wants to use such a result for the study of Volterra semimartingales of the form $X_t = \int_0^t \Psi_{t,s} \, dS_s$, $t \geq 0$, the classic assumption that one has a fixed measure η is too restrictive; the mixture over the integrands needs to be taken instead with respect to a stochastic kernel on the parameter space. To handle that situation and prove a corresponding new stochastic Fubini theorem, we introduce a new notion of measure-valued stochastic integration with respect to a general multi-dimensional semimartingale. As an application, we show how this allows to handle a class of quite general stochastic Volterra semimartingales.

Joint work with Tahir Choulli.

Malkhaz Shashiashvili (I. Javakhishvili Tbilisi State University)

Sensitivity analysis of the early exercise boundary with respect to variations of the local volatility

We study the stability problem of an early exercise boundary with respect to local volatility variations for the American put option in the generalized Black-Sholes model. Some estimates of the sensitivity of the early exercise boundary have already been given in Achdou [1], but under additional regularity assumptions (see Proposition 4.4 there) on the time derivative of the value function and on the early exercise boundary. We develop different analytical approach which is inspired by the paper Shashiashvili [2], where an estimate of the distance in variation between the predictable components of two Snell envelopes is given. We establish an estimate of the area between the early exercise boundaries for the American put option with different local volatility functions through the uniform norm between the volatilities. Moreover the constant factor on the right-hand side of the estimate is given in explicit form (see Theorem 2.1 in Shashiashvili et al [3]).

Joint work with Besarion Dochviri and Omar Purtukhia.

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Grigol Sokhadze (I. Javakhishvili Tbilisi State University)

About Bernoulli type regression

Let Y is a random element, which takes two values 1 and 0 with probabilities p and $1-p$. Assume that the probability of "success" p is a function of an independent variable

$$x \in [0;1]: \quad p = p(x) = P\{Y = 1 \mid x\}.$$

Let x_i, $i = 1, 2, \dots, n$, be the division points of the interval $[0;1]$ which are chosen from the relation

$$H(x_i) = \int_0^{x_i} h(x)\,dx = \frac{2i-1}{2n}\,, \quad i = 1, 2, \dots, n.$$

where $h(x)$ is the known positive distribution density on $[0;1]$.

Let further Y_i, $i = 1, \dots, n$, be independent Bernoulli random variables with $P\{Y_i = 1 \mid x_i\} = p(x_i)$, $P\{Y_i = 0 \mid x_i\} = 1 - p(x_i)$, $i = 1, \dots, n$.

The problem consists in estimating the function $p(x)$, $x \in [0;1]$, by the sampling $Y_1, \dots, Y_n$ and investigation some asymptotical properties ([1-4]).

Joint work with Elizbar Nadaraya and Petre Babilua.

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Winfried Stute (University of Giessen, Germany)

Principal component analysis of the Poisson process

The Poisson process constitutes a well-known model for describing random events over time. It has many applications in marketing research, insurance mathematics and finance. Though it has been studied for decades not much is known how to check the validity of the Poisson process. In this talk we present the principal component decomposition of the Poisson process which enables us to derive finite sample properties of associated goodness-of-fit tests.

Joint work with Maria Bianca Popescu.

Vaja Tarieladze (Georgian Technical University)

Some probabilistic results of N. Vakhania

In this talk, which is based on [1], we plan to discuss some probabilistic results of Professor Nicholas Vakhania (1930-2014).

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Nizar Touzi (Ecole Polytechnique)

Viscosity solutions of path-dependent nonlinear PDEs

Motivated by recent developments in non-Markovian stochastic control and stochastic differential games, a notion of viscosity solutions for path-dependent PDEs was recently introduced as a substitute to the Sobolev-type solutions provided by backward SDEs and their second order extension. Similar to the Crandall-Lions notion of viscosity solutions, the main idea is the use of "tangent paraboloids" as test functions. However, the path-dependent definition replaces the pointwise tangency by a tangency in mean, thus allowing for a wider class of test functions. Consequently, this definition has a potential for an easier uniqueness argument. We provide the most recent well-posedness results. In particular, uniqueness is a consequence of the comparison result with a purely probabilistic proof.

Davit Varron (University of Franche-Comté)

A Donsker and a Glivenko-Cantelli theorem for a class of radom measures generalizing the empirical process

We establish a Glivenko-Cantelli and a Donsker theorem for sequences of random discrete measures which generalize the empirical measure, under conditions on uniform entropy numbers that are common in empirical processes theory. Those measure have the following general representation $P_n := \sum_{i \geq 1} p_{i,n} \delta_{X_{i,n}}$ where both the infinitely may locations and weights are random. Some illustrative applications in nonparametric Bayesian theory and randomly sized sampling are provided: we recover and slightly improve well known posterior consistency/inconsistency and Bernstein von Mises theorems in strong topologies, for posterior distributions of the Poisson-Dirichlet process. We also derive interesting criteria to prove first and second order convergences of random sequences of normalized homogenous completely random measures.

Participants

Name	Affiliation
Hansjoerg Albrecher	University of Lausanne
Petre Babilua	Tbilisi State University
Philippe Berthet	Toulouse Mathematics Institute
Umut Can	University of Amsterdam
Besarion Dochviri	Tbilisi State University
John Einmahl	Tilburg University
Hans-Jürgen Engelbert	Friedrich Schiller University of Jena
Dietmar Ferger	Technische Universität Dresden
Eva Ferreira	University of the Basque Country
Irène Gijbels	KU Leuven
Shota Gugushvili	Leiden University
Estate Khmaladze	Victoria University of Wellington
Roger Koenker	University of Illinois at Urbana-Champaign
Hira L. Koul	Michigan State University
Roger Laeven	University of Amsterdam
Tapabrata Maiti	Michigan State University
Badri Mamporia	Georgian Technical University
Michael Mania	A. Razmadze Mathematical Institute
Gennadi Martynov	IITP RAS & HSE
Ian McKeague	Columbia University
Robert Mnatsakanov	West Virginia University
Elizbar Nadaraya	Tbilisi State University
Goran Peskir	The University of Manchester
Omar Purtukhia	Tbilisi State University, A. Razmadze Mathematical Institute
Holger Rootzén	Chalmers University
Johannes Schmidt-Hieber	Leiden University
Martin Schweizer	ETH Zürich
Malkhaz Shashiashvili	Tbilisi State University
Grigol Sokhadze	Tbilisi State University
Winfried Stute	University of Giessen
Revaz Tevzadze	Georgian Technical University
Vaja Tarieladze	Georgian Technical University
Nizar Touzi	École Polytechnique
Davit Varron	University of Franche-Comté