A NON-ABELIAN GENERALIZATION OF VILENKIN SYSTEMS

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The study of Walsh series should be performed by representing the Walsh functions as the characters of the dyadic group, i.e., the complete product of the discrete cyclic group of order 2 with the product of topologies and measures. Vilenkin in 1947 generalized this structure studying the complete product of arbitrary cyclic groups. In [1] the authors generalized the Vilenkin systems. The main idea is to take the complete direct product of arbitrary finite groups, even non-abelian groups. The characters of a finite non-abelian group can not form a complete system because their number is less than the order of the group. The missing functions can be obtained by computing the representations of the group. Product systems formed by characters and normalized coordinate functions of these representations are called representative product systems.

Results obtained for representative product systems can differ considerably if they are defined on non-abelian groups (see [2]). Nonetheless, for specific cases we obtain the same result. For instance, we obtain positive results with respect to the convergence in L^p-norm $(1 < p < \infty)$ of Fourier series with respect to certain representative product systems defined on the bounded complete product of quaternion groups.

References

[1] G. Gát, R. Toledo, *L^p-norm convergence of series in compact totally disconnected groups*, Anal. Math. **22** (1996), 13–24.
[2] R. Toledo, *Negative results concerning fourier series on the complete product of $\mathcal{S}_3$*, J. Inequal. Pure and Appl. Math. **9(4), Art. 99** (2008), 7 pp.

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